Specific Accreditation Guidance

Infrastructure and Asset Integrity

Measurement Uncertainty in Geotechnical Testing

January 2018

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Measurement Uncertainty for Geotechnical testing

This document is intended as a Guide for Geotechnical and Civil Construction Materials Testing Laboratories to assist them in the estimation of measurement uncertainty (MU).

There is a number of approaches that could be used to determine MU. This guide outlines only one approach using indicative data. It is the responsibility of each laboratory to ensure that the approach and data used to calculate MU is both accurate and appropriate for its testing circumstances.

This guide is provided on the basis that it is not definitive and may contain some information which differs from practice performed in some laboratories. It is intended that this guide be read in conjunction with the NATA publication *Assessment of Uncertainties of Measurement for Calibration and Testing Laboratories* by RR Cook.

The examples will need to be evaluated by the laboratory when estimating a particular MU to take into account the unique circumstances relating to the individual laboratory.

What Is Measurement Uncertainty?

Measurement uncertainty is associated with a result, e.g. moisture content, and defines the range of values which could reasonably be attributed to that result.

It is about the measuring process. Incorrectly following a procedure, gross errors or mistakes are not covered by measurement uncertainty.

Simplified Approach

A simplified approach has been taken to the estimation of measurement uncertainty in this series of examples.

This approach is summarised as:

Step 1 – Determine what is being measured.

Step 2 – Determine the sources of uncertainty in obtaining the test result.

Step 3 – Assign a value for each source of uncertainty ***U value*** and assign the type of distribution for each uncertainty.

Step 4 – Calculate the standard uncertainty $u(x)_i$ for each component (U).

Step 5 – Determine a weighting for each component c_i.

Step 6 – Calculate the final value for each component by multiplying the standard uncertainty $u(x)_i$ by the weighting factor c_i to give this value $u_i c_i$. This value is then squared to express in the terms of a variance $u_i(y)^2$.

Add the individual values of the variance value for each component to give $\Sigma u_i(y)^2$.

Calculate the square root of the sum of the variances of each component to give ***u(y)***.

Step 7 – From the combined uncertainty determine the expanded uncertainty which is then reported with the test result.

The examples shown in this guide will highlight areas of the spreadsheet in different colours corresponding to each of the seven steps.

Step 1 - What is being Measured?

Examine the test method. Look for direct sources of uncertainty and any implied sources due to technique, environment, etc.

What is being measured is defined usually in the name of the test method and by the final result that is required in the reporting clauses, e.g. moisture content, field density, compressive strength.

Take note of any restrictions or limitations that explain how the sample is treated. In the moisture content example given in this guide, the material is a fine soil; therefore, the size of the sample should be greater than 30 g. The balance used must have a limit of performance of ± 0.5 g or better.

In this case, the same balance and sample container are used throughout the test.

The equation required to calculate the test result is the starting point when investigating contributions to the overall uncertainty. This record must be included with the documentation of the measurement uncertainty estimation.

For example, for the moisture content determination in AS 1289.2.1.1 for fine material:

Moisture content (***w***%) = [(wet mass – dry mass)/dry mass] x 100.

Step 2 - Sources of Uncertainty

This is an important step which must be investigated and needs to be taken for all tests whether the reporting of MU is required or not.

(a) The first part of this identification process involves going through the test method and charting each step and listing the actual measurements to be taken.

(b) Next, review the listing of the sources of uncertainty. If available, what will affect each step?

For example:

- the accuracy of measurement using a balance (limit of performance is an extreme estimate);
- how well can I read the equipment ?;
- how well is this defined in the method (e.g. a thermometer readable to 1°C) ?; and
- what are the inherent problems, e.g. constant mass, inability to completely remove air from sample in a pycnometer?

If the test method indicates how measurements or readings are to be rounded, this is not considered a source of uncertainty. If the test method does not specify the rounding and you need to round then this is considered a source of uncertainty.

Be aware that the estimation of the MU does not take into account gross operator errors or mistakes. It is important to follow the method so that such errors do not affect the final results.

When data derived from one test is used in another, then the results of the first test will have an uncertainty which in turn needs to be included in the MU estimate for the second test.

When test data is correlated against other data and a correction is to be made, e.g. correlation of subsidiary moisture content methods, the correction to be applied needs to be made prior to performing the measurement uncertainty estimate. Any uncertainty in the correlation needs to be taken into account, e.g. the standard error of a regression.

For example, when determining the moisture content of a test specimen, the following measurements are taken:

- Mass of the container (m_a)
- Mass of the container plus wet sample (m_b)
- Mass of the container plus dry sample (m_c).

i.e. Moisture content ($w\%$) = $[(m_b - m_c)/(m_c - m_a)] \times 100$

This is not the complete story as other factors (i.e. the environment, equipment operation, etc.) influence the measurements which contribute to the overall measurement uncertainty. It has been assumed that the test specimen has been collected, transported, stored and prepared correctly, i.e. has not dried out.

These sources of uncertainty may include:

- Accuracy of the balance used;
- Convection currents that may affect the dry mass if a warm sample is weighed, particularly on an analytical balance;
- Absorption of moisture during cooling;
- Achieving constant mass; and
- Weighing precision – due to flicker (from say 22.77 to 22.79 g).

The uncertainty associated with each of these is represented by u_i. Thus the uncertainties are:

- u_b Accuracy of the balance used
- u_w Convection currents that may affect the dry mass
- u_a Absorption of moisture during cooling
- u_{cm} Achieving constant mass
- u_p Weighing precision

These can be shown in the equation:

Moisture content
$= [\{(\boldsymbol{m}_\mathbf{b} \pm \boldsymbol{u}_\mathbf{b} \pm \boldsymbol{u}_\mathbf{p}) - (\boldsymbol{m}_\mathbf{c} \pm \boldsymbol{u}_\mathbf{b} \pm \boldsymbol{u}_\mathbf{w} \pm \boldsymbol{u}_\mathbf{a} \pm \boldsymbol{u}_\mathbf{cm} \pm \boldsymbol{u}_\mathbf{p})\}/\{(\boldsymbol{m}_\mathbf{c} \pm \boldsymbol{u}_\mathbf{b} \pm \boldsymbol{u}_\mathbf{w} \pm \boldsymbol{u}_\mathbf{a} \pm \boldsymbol{u}_\mathbf{cm} \pm \boldsymbol{u}_\mathbf{p}) - (\boldsymbol{m}_\mathbf{a} \pm \boldsymbol{u}_\mathbf{b} \pm \boldsymbol{u}_\mathbf{p})\}] \times 100$

Step 3 - Estimates of Uncertainty and Types of Distribution

Each source of uncertainty needs to have a range estimated and its probability of occurrence (distribution) determined (see Note 1 for determining the distribution).

Estimates can be derived from specified equipment calibration certificates and from the test method itself. Alternatively, the maximum values can be obtained from the specified values in the test method for this equipment.

Other uncertainties relate to the capability of reading measuring instruments, e.g. meniscus readings, reading between markings on a dial gauge.

Test methods also may quantify certain sources of uncertainty, for example:

- AS 1289.2.1.1 – constant mass – 0.1% of total mass of moist sample;
- AS 1141.11.1 - not more than 1% of mass retained on sieve shall pass after 1 min of shaking.

Uncertainty may need to be calculated from the tolerances provided in other standards or test methods, e.g. the tolerances on the dimensions of sieves taken from AS 1152, or from the differences in aperture openings determined from annual measurement checks.

In some cases, the laboratory may need to perform additional testing to gain measurement uncertainty data, e.g. absorption and thermal currents,. In particular, this may be needed when a property is dependent on another measured value which is not included in the equation, e.g. relationship between CBR and moisture content.

When the uncertainty has been calculated from another test or obtained from a calibration certificate, it is calculated by dividing the reported expanded uncertainty by the reported coverage factor (k) which is usually included in a calibration report (e.g. **k**=2) (see also step 7).

How large are these uncertainties?

Using the above example of moisture content:

The accuracy of the balance can be obtained from either the calibration report or from the limit of performance (LoP) shown in the test method. E.g. **LoP** ±0.05, gives range of 0.1g (i.e. 2 x **LoP**). Therefore $\pm \boldsymbol{u}_\mathbf{b}$ = 0.05 g. In this test, the same balance is used for all three weighings. The distribution is rectangular with a factor of $\sqrt{3}$ (see Note 1).

The effect of convection is again very small. An estimate of this effect u_w is 0.004 g (i.e. about 0.01% of the dry material). This could range from 0 to 0.004g (i.e. u_w = ±0.002g) and the distribution is rectangular with a factor of √3 (see Note 1).

The absorption of water is usually very small. An uncertainty of 0 to 0.01g could be estimated (i.e. about 0.04% of the dry material). u_a = 0.01/2 = ±0.005g and the distribution is rectangular with a factor of √3 (see Note 1).

Achieving constant mass – AS 1289.2.1.1 indicates that the mass loss between subsequent weighings shall not be greater than 0.1% of the initial wet mass. The value of 30.9 g for the wet sample obtained in the test example would give a worst case estimate of 0.0309 g. As this value could range equally from 0 to 0.0309 g, then u_{cm} = 0.015 g and the distribution is rectangular with a factor of √3 (see Note 1).

There is uncertainty in the weighing precision for the balance as the values may change from say 22.77 to 22.79, the most likely value being the recorded value of 22.78. Estimate of this uncertainty u_p = ± 0.01 g. In this case. the uncertainty value is from a triangular distribution. (see Note 1) and a factor of √6.

In practice u_w and u_a could be left out as they might be considered to have negligible effect on the overall MU. For this example they have been included.

The sources of uncertainty then become:

m_a	→	u_b	→	0.05	factor	=	√3
	→	u_p	→	0.01	factor	=	√6
m_b	→	u_b	→	0.05	factor	=	√3
	→	u_p	→	0.01	factor	=	√6
m_c	→	u_b	→	0.05	factor	=	√3
	→	u_w	→	0.002	factor	=	√3
	→	u_a	→	0.005	factor	=	√3
	→	u_p	→	0.01	factor	=	√6

Step 4 - Calculating Standard Uncertainties

If each uncertainty could be measured, a range of values would be obtained. As this cannot be done in most cases, statistics can be used to determine an estimate of the spread. This is called the standard uncertainty.

The standard uncertainty is calculated by dividing the uncertainty value by the factor related to the type of distribution from which it came. This factor is determined in Step 3 above (see also Note 1).

m_a	→	u_b	→	0.05	factor	=	√3	$u(x)_i$	=	0.028868
		u_p	→	0.01	factor	=	√6	$u(x)_i$	=	0.004082
m_b	→	u_b	→	0.05	factor	=	√3	$u(x)_i$	=	0.028868

		u_p	→	0.01	factor	=	√6	$u(x)_i$	=	0.004082
m_c	→	u_b	→	0.05	factor	=	√3	$u(x)_i$	=	0.028868
	→	u_w	→	0.002	factor	=	√3	$u(x)_i$	=	0.001155
	→	u_a	→	0.005	factor	=	√3	$u(x)_i$	=	0.002886
	→	u_{cm}	→	0.015	factor	=	√3	$u(x)_i$	=	0.086605
	→	u_p	→	0.01	factor	=	√6	$u(x)_i$	=	0.004082

Note: A combined uncertainty for all the components related to determining the dry mass m_c could be calculated, i.e. $m_c \pm u_c$, where $u_c = \sqrt{(u_b^2 + u_w^2 + u_a^2 + u_{cm}^2 + u_p^2)}$ provided each uncertainty is expressed in grams.

Step 5 - Weighting

At this point in the process you need to determine a weighting to each uncertainty. The process employed should allow the uncertainty to be expressed in the same units as the result to be reported.

Where a simple one-to-one relationship between a change to a measured value and a change in the final value occurs, a weighting of 1 can be applied to the standard uncertainty.

For example, if the perimeter of a brick is measured in mm, an uncertainty in the width dimension of, say 0.1 mm, will produce a direct change in the total length equal to the uncertainty in the width measurement.

If you are determining the area of a circle by measuring the diameter, there is not a direct one-to-one relationship between the diameter measurement and the area.

$\mathbf{A} = \boldsymbol{\pi}\mathbf{d}^2/4$. A change of 1 per cent in the diameter will cause a 2% change in the area. Thus the weighting will be 2.

Obtaining Weighting Factors (ci) - Numerical Calculation Using Small Increments

In the examples, a simplified method of calculating the weighting factor for each uncertainty u_i is used.

(a) Enter a set of data inputs and calculate the result.

(b) Add a small increment to just one of the input values while keeping the remaining inputs constant. Recalculate the result.

(c) Subtract the result obtained in (b) from the first result obtained in (a). Divide this value by the small increment used.

(d) Repeat this process (a) to (c) for each measurement.

For the moisture content determination, example 1, a laboratory determines both the wet and dry mass of a test specimen as shown below.

Step (a)
Moisture content ($w\%$) = $[(m_b - m_c)/(m_c - m_a)] \times 100$
Mass of container (m_a) = 22.78 g
Mass of container plus wet sample (m_b) = 53.68 g
Mass of container plus dry sample (m_c) = 47.92 g

Moisture content (w%) = 100*(53.68 – 47.82)/(47.92-22.78) = 22.91169 %

Step (b)
With a small increment of 0.01g added to the mass of the container plus wet sample, calculate the new mass of container plus wet sample, i.e. 53.68 + 0.01 = 53.69. Then calculate the new moisture content, i.e. 22.95417 %

Step (c)
The difference in moisture contents is 22.91169 – 22.95147 = -0.03978

The calculated weighting is then -0.03978/0.01 = -3.978

Step (d)
Similarly, the weighting for the same small increment for the m_b is -4.887 and m_c is 0.912.

See attached spreadsheet in Example 1.

Step 6 - Calculations

For each uncertainty component calculate the value $u_i c_i$ by multiplying the standard uncertainty $u(x)_i$ by the weighting factor c_i.

Square these values of $u_i c_i$ to express in them in terms of a variance $u_i(y)^2$.

Add the individual variance values for each component to give the sum of the variances $\Sigma u_i(y)^2$.

Calculate the square root of the sum of the variances of each component $u(y)$ which is the combined uncertainty for the test result.

For the moisture content, example 1, see the attached spreadsheet for these calculations giving a value of combined uncertainty for the test result of 0.47204.

Step 7 – Expanded Uncertainty (Mu) and Reporting

Multiply the combined uncertainty by a coverage factor to give the expanded uncertainty which is then reported with the test result.

The coverage factor for this simplified approach to estimating MU is generally taken as 2. More detailed calculations involving degrees of freedom are required to obtain different coverage factors should a more accurate estimate of MU be required.

When reporting of uncertainty is required the following should be included in the report:

- the test result to the appropriate number of significant figures or as directed in the test method;
- ± the measurement uncertainty estimated in the same units and significant figures as the test result;
- the coverage factor used and whether a nominal value has been used or it has been calculated;
- a statement such as “The uncertainty of measurement value shown does not include any estimate of the effects associated with sampling”; and
- an additional statement such as “Test results should be assessed using precision in terms of repeatability and reproducibility, measurement uncertainty and sampling effects”, could be added.

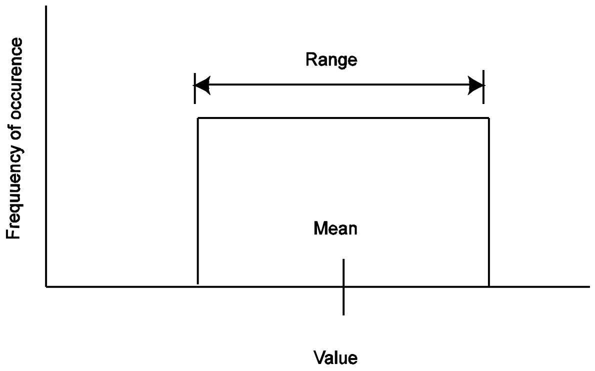
NOTE 1 - Types of Distribution and Degrees of Freedom

Each source of uncertainty needs to have its probability of occurrence (distribution) determined. In many cases an estimate must be based on experience. Having estimated a range for a particular uncertainty, the chance of any particular value occurring needs to be estimated

Note: In this simplified approach, the degrees of freedom have not been taken into account. When more rigorous estimations of uncertainty are required, the degrees of freedom must be considered

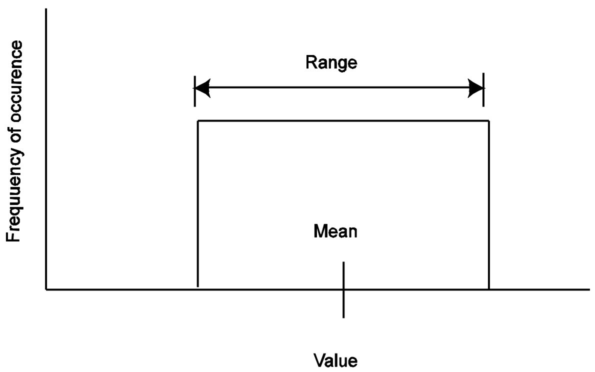
Rectangular Distribution, Factor = √3

Although a magnitude (range) is estimated for a particular uncertainty, sometimes it is not known where in this range a particular uncertainty value might occur. In other words without any more knowledge the chance of being any particular value in the range is the same. In this case, it is reasonable to treat the distribution as rectangular



Range R Standard Deviation = (R/2) / √3

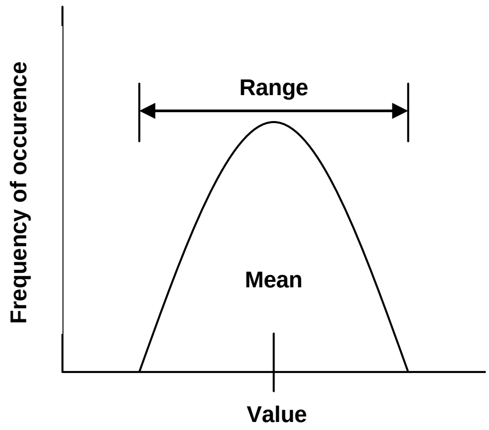
For example, with the determination of moisture content example, the uncertainty associated with the absorption of moisture cannot be measured, but the range is estimated to be between 0 g and 0.01 g. The actual value lies somewhere between these two values. The probability that it is 0.005 is the same as the value being 0.009



Range R = 0.01 Standard Deviation = (0.01/2) /√3 = 0.002887

Normal Distribution, Factor = √2

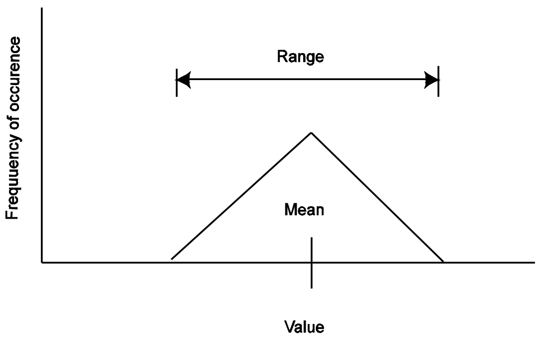
If the source of uncertainty can be measured, e.g. the accuracy of a balance, repeat determinations, etc., then a normal distribution is reasonable



Range R Standard Deviation = (R/2) / √2

Triangular Distribution, Factor = √6

The triangular distribution can be used when there is more knowledge and confidence about the chance of the uncertainty occurring than in the case of a rectangular distribution, but less than for a normal distribution



Range = R Standard Deviation = (R/2)/ √6

An example of this might be an operator's ability to read a pressure gauge.

The uncertainty associated with reading the gauge is small, ± 1 division. It is a gauge with a mirrored back and a fine pointer. It is mounted to assist the operator to read it accurately. Based on this, it is more likely that the uncertainty is closer to zero than 1 division. In this case a triangular distribution is appropriate.

As the same pressure cannot be reproduced consistently, repeated readings are not possible and a normal distribution cannot be considered

Example 1 - Moisture Content - AS 1289.2.1.1

Step 1: What is being measured?

The moisture content of soil is determined in accordance with AS 1289 2.1.1 for fine soil.

The calculation is moisture content w% = 100 * ((Initial mass - Dry mass)/Dry mass).

Step 2: Sources of Uncertainty

Uncertainty occurs each time a weighing is done due to accuracy of the balance and the precision of reading for each of the following.

- Mass of the container ($\boldsymbol{m}_a$)
- Mass of the container plus wet sample ($\boldsymbol{m}_b$)
- Mass of the container plus dry sample ($\boldsymbol{m}_c$)

i.e. Moisture content ($\boldsymbol{w}$%) = $[(\boldsymbol{m}_b - \boldsymbol{m}_c)/(\boldsymbol{m}_c - \boldsymbol{m}_a)] \times 100$

A sample of at least 30 g is required. A balance with a limit of performance not exceeding ± 0.05g must be used.

The same container and balance is used for each weighing throughout the test.

Other factors (i.e. the environment, equipment operation, etc.) influence the measurements which contribute to the overall measurement uncertainty. It has been assumed that the test specimen has been collected, transported, stored and prepared correctly, i.e. has not dried out.

The sources of uncertainty are:

- The accuracy of the balance ($\boldsymbol{u}_b$)
- Convection currents that may affect the dry mass if a warm sample is weighed, particularly on an analytical balance ($\boldsymbol{u}_w$)
- Absorption of moisture during cooling ($\boldsymbol{u}_a$)
- Achieving constant mass ($\boldsymbol{u}_{cm}$)
- Precision of reading the balance ($\boldsymbol{u}_p$)

Moisture content = $[\{(\boldsymbol{m}_b \pm \boldsymbol{u}_b \pm \boldsymbol{u}_p) - (\boldsymbol{m}_c \pm \boldsymbol{u}_b \pm \boldsymbol{u}_w \pm \boldsymbol{u}_a \pm \boldsymbol{u}_{cm} \pm \boldsymbol{u}_p)\}/\{(\boldsymbol{m}_c \pm \boldsymbol{u}_b \pm \mathbf{u}_w \pm \boldsymbol{u}_a \pm \boldsymbol{u}_{cm} \pm \boldsymbol{u}_p) - (\boldsymbol{m}_a \pm \boldsymbol{u}_b \pm \boldsymbol{u}_p)\}] \times 100$

Step 3: Estimates of Uncertainty and Types of Distribution

For the purpose of the example, the uncertainty related to measurement of mass is taken from the actual value of the Limit of Performance shown on the calibration certificate.

The error due to convection currents $\boldsymbol{u}_w$ is estimated to be in the range ± 0.01g.

The error due to absorption $\boldsymbol{u}_a$ is estimated to be in the range ± 0.01g of the total dry sample.

The maximum error related to constant mass is taken from the difference between the last two weighings which meet the constant mass requirement, in this case $\boldsymbol{u}_{cm}$ is in the range ± 0.015 g.

The error due to weighing precision is taken from the changing of values when reading and in this example is taken as 0.01g.

Distributions

As the value of the limit of performance of the balance is taken from the test method, it can be considered to come from a rectangular distribution with a factor of square root of 3.

As the constant mass, absorption and convection current values could be any value between zero and the maximum value detailed here, the distribution is rectangular and thus a factor of square root of 3 is used.

The error due to weighing precision can be considered to come from a triangular distribution with a factor of square root of 6.

Step 4: Calculating Standard Uncertainties

Standard uncertainties are calculated by dividing the uncertainties by the factor corresponding to the distribution used.

See the attached spreadsheet, column I shown in green.

Step 5: Weighting

The weighting calculation is shown in columns G, H and I (shown in turquoise in the example) using small increments in each of the mass determinations. Each mass is separately varied by a small increment to see its effect on the moisture content. This calculated value is then divided by the small increment to give the weighting.

Step 6: Calculations

For each uncertainty component calculate, the final input value ($\boldsymbol{u}_i\boldsymbol{c}_i$) by multiplying the standard uncertainty $\boldsymbol{u(x)}_i$ by the weighting factor $\boldsymbol{c}_i$.

Square these values of ($\boldsymbol{u}_i\boldsymbol{c}_i$) to express them in terms of a variance $\boldsymbol{u}_i\boldsymbol{(y)}^2$.

Add the individual values of the variance value for each component to give the sum of the variances $\Sigma u_i(y)^2$.

Calculate the square root of the sum of the variances of each component ***u(y)*** which is the combined uncertainty for the test result.

Step 7: Expanded Uncertainty (Mu) and Reporting

A coverage factor of 2 is selected for the rigour required for this type of test. The combined uncertainty calculated in Step 6 is multiplied by the coverage factor to give the expanded uncertainty. Finally the test result and uncertainty value to be reported are shown on the spreadsheet.

A	B	C	D	E	F	G	H	I	J	K	L
Source of error	Distribution	Error	Factor	Standard uncertainty	Weighting	Calculations	Coverage factor	Expanded uncertainty	Report	Data Entry	

Test Parameters	*Symbol*	*Test Data*	*Increment*	Weighting Incremental Calculations			
				soil/wet cont	m_b	m_c	*soil/dry cont*
Mass of Container (g)	m_a	22.78	0.01	22.79	22.78	22.78	22.78
Mass of Container plus wet soil (g)	m_b	53.68	0.01	53.68	53.69	53.68	53.68
Mass of Container plus dry soil (g)	m_c	47.92	0.01	47.92	47.92	47.93	47.92
	m_a	22.78	0.01	22.78	22.78	22.78	22.79
Moisture Loss (g)		5.76		5.75	5.77	5.75	5.76
Dry Soil (g)		25.14		25.13	25.14	25.15	25.13
Moisture content (%)		22.9116945		22.8810187	22.9514718	22.86282306	22.920812
Rounded Result (%)		23.0					
			Difference in MC %	0.030675808	-0.03977725	0.048871449	-0.0091173
			Weighting	3.067580799	-3.97772474	4.887144911	-0.9117268

Measurement	*Source*	*Uncertainty*	*Distribution Type*		*U* value (g)	*Dist Factor*	*Std Uncertainty* $u(x)_i$	*Weighting* c_i	$u_i c_i$	$(u_i(y))^2$
Mass of container (wet soil)	balance	u_b	rectangular		0.04	1.73205081	0.023094011	3.0675808	0.0708427	0.0050187
	reading	u_p	triangular		0.001	2.44948974	0.000408248	3.0675808	0.0012523	1.568E-06
Mass of wet soil+container	balance	u_b	rectangular		0.04	1.73205081	0.023094011	-3.9777247	-0.0918616	0.0084386
	reading	u_p	triangular		0.001	2.44948974	0.000408248	-3.9777247	-0.0016239	2.637E-06
Mass of dry soil+container	balance	u_b	rectangular		0.04	1.73205081	0.023094011	4.8871449	0.1128638	0.0127382
	conv currents	u_w	rectangular		0.002	1.73205081	0.001154701	4.8871449	0.0056432	3.185E-05
	absorption	u_a	rectangular		0.005	1.73205081	0.002886751	4.8871449	0.014108	0.000199
	const mass	u_{cm}	rectangular		0.015	1.73205081	0.008660254	4.8871449	0.0423239	0.0017913
	reading	u_p	triangular		0.001	2.44948974	0.000408248	4.8871449	0.0019952	3.981E-06
Mass of container dry soil)	balance	u_b	rectangular		0.04	1.73205081	0.023094011	-0.9117268	-0.0210554	0.0004433
	reading	u_p	triangular		0.001	2.44948974	0.000408248	-0.9117268	-0.0003722	1.385E-07
									SUM	0.0282259
									$u_c(y)$	0.1680055
									k	2
									U_{95}	0.34

REPORT:	**Moisture Content**		**23.0**	%
	with an expanded measurement uncertainty of	±	**0.3**	%
	at a confidence level of		**95**	%
	with a nominal coverage factor of		**2**	

The uncertainty of measurement value shown does not include any estimate of the effects associated with sampling. Test results should be assessed using precision in terms of repeatability and reproducibility, measurement uncertainty and sampling effects.

Example 2 - Compressive Strength of Concrete - AS 1012.9

Step 1: What is being measured?

The measurement uncertainty is based on the calculation shown in AS 1012.9 and the use of 100 mm diameter cylinders which meet the height requirements, i.e. height to diameter ratio of 1.95 to 2.05.

Compressive Strength (***UCS***) = Applied Load (***P***) /Cross Sectional Area (***A***) of the cylinder.

$\boldsymbol{A} = \pi \times (\text{diameter } ((\boldsymbol{d}_1 + \boldsymbol{d}_2)/2))^2/4$

where d = the diameter of the cylinder (measurements at two points 1 and 2).

A compression testing machine with an analogue (pointer and scale) indicator readout which can be read to about 0.5 kN was considered.

For the purpose of this example, the uncertainties associated with measuring the compressive strength of a 25 MPa concrete using a standard 100 mm diameter cylinder is considered.

Step 2: Sources of Uncertainty

Uncertainties associated with the applied load include:

- Uncertainty of the measured load ($\boldsymbol{u}_p$) from specification of the testing machine, Class A = ±1% of the measured load range; (in this example a range = 1 x 193 / 100 = ± 1.93 kN). This could be anywhere within the range and thus a rectangular distribution.

- Uncertainty associated with reading the display of the testing machine ($\boldsymbol{u}_m$), a range of ± 0.5 kN. This could be anywhere within the range, and thus a rectangular distribution.

- Uncertainty due to eccentric centring of cylinder in testing machine ($\boldsymbol{u}_\mathbf{e}$), say about ± 1 mm. It is estimated that this could amount to a decrease in load of a range of ± 0.1% of the applied load (in this case a range of ± 0.1 x 193 / 100 = ± 0.193 kN). This could be anywhere within the range and thus a rectangular distribution.

- Uncertainty due to angle of cap ($\boldsymbol{u}_\mathbf{cap}$), maximum of 3° from test method, say ± 1.5%.

i.e. the vertical load applied to the cylinder is ***P*** x cos 1.5°, therefore the uncertainty is the measured load - the vertical load applied to the cylinder.

$\boldsymbol{u}_\mathbf{cap} = \boldsymbol{P} - \boldsymbol{P} \times \cos 1.5° = \boldsymbol{P} \times (1 - \cos 1.5°) = 0.00034 \times \boldsymbol{P} = 0.00034 \times 193$

i.e. $\boldsymbol{u}_{cap}$ is in the range ± 0.06562 kN

This could be anywhere within the range and thus a rectangular distribution is appropriate.

- Uncertainty due to slenderness ratio ($\boldsymbol{u}_{\mathbf{s}}$), i.e. ratio of length to diameter. In the case of standard cylinders this value is 2 and any slight varition outside the limits is negligible. On this basis, the uncertainty is taken as 0. For examples with cores, this uncertainty would need to be taken into account if the slenderness ratio is not 2.

Uncertainty due to measurement of diameters:

Uncertainty associated with measuring each diameter ($\boldsymbol{u}_{\mathbf{d1}}$ and $\mathbf{u}_{d2}$) of the dial gauge will depend on the precision to which the measuring device is read ($\boldsymbol{u}_{r1}$ and $\boldsymbol{u}_{r2}$) and the accuracy of the measuring device ($\boldsymbol{u}_{g1}$ and $\boldsymbol{u}_{g2}$) itself. The test method states measurement to within 0.2 mm, therefore uncertainty could be considered anywhere in the range ± 0.1 mm for each measured diameter, and thus a rectangular distribution. A smaller uncertainty could be estimated based on the uncertainty shown on the calibration report for the measuring device.

For the purpose of this example, the uncertainty due to the calibration of the dial gauge or vernier is considered to be small in relation to the reading to the nearest 0.2 mm and is not included in the calculations, i.e. $\boldsymbol{u}_{d(n)} = \boldsymbol{u}_{r(n)}$.

Uncertainty associated with variation in load rate

An additional uncertainty in the strength can be due to the rate of loading ($\boldsymbol{u}_{\mathbf{l}}$). This uncertainty is added to the total uncertainty and is not included in the uncertainty of applied load. Data for this can be obtained from research papers and this value may differ as the compressive strength changes. An estimate of a range of ± 0.1% of the compressive strength is considered reasonable for the 20 ± 2 kN/min specified in the test method. Therefore for 25 MPa concrete, 0.1% is equal to ± 0.025 MPa and is a rectangular distribution.

Uncertainty associated with the constant, π

Depending on the amount of decimal points used in the calculation, this uncertainty ($\boldsymbol{u}_{pi}$) may be significant. E.g. , using a value of 3.142 may introduce an uncertainty compared to the significant figures used by a spreadsheet, in this example, is 3.142 - 3.1416 = 0.0004. This will depend on the method used for the calculation.

Uncertainty expressed in the equation are then:

$$\boldsymbol{UCS} = [(\boldsymbol{P} \pm \boldsymbol{u}_{\mathbf{p}} \pm \boldsymbol{u}_{m} \pm \boldsymbol{u}_{\mathbf{e}} \pm \boldsymbol{u}_{\mathbf{cap}} \pm \boldsymbol{u}_{\mathbf{s}})/ ((\pi \pm \boldsymbol{u}_{pi}) \times ((\boldsymbol{d}_{1} \pm \boldsymbol{u}_{\mathbf{d1}} + \boldsymbol{d}_{\mathbf{2}} \pm \boldsymbol{u}_{\mathbf{d2)}})/2)^{2)}/4] \pm \boldsymbol{u}_{\mathbf{l}}$$

Step 3: Estimates of Uncertainty

The basis of the estimates of uncertainty and the estimate of value are shown above:

$\boldsymbol{u}_{p}$ - the uncertainty of the measured load taken as ± 0.5 kN - rectangular distribution

$\boldsymbol{u}_m$ - the uncertainty associated with reading the testing machine ± 0.5 kN - rectangular distribution

$\boldsymbol{u}_e$ - the uncertainty due to eccentric centring taken as ± 0.193 kN - rectangular distribution

$\boldsymbol{u}_{cap}$ - the uncertainty associated with the angle of loading = ± 0.0652 kN - rectangular distribution

$\boldsymbol{u}_s$ - the uncertainty associated with the slenderness ratio = 0.

$\boldsymbol{u}_{pi}$ - the uncertainty of the constant pi due to rounding = ± 0.0004 - rectangular distribution

$\boldsymbol{u}_{d1}$ -the uncertainty associated with the measurement of diameter 1 = ± 0.1 mm - rectangular distribution

$\boldsymbol{u}_{d2}$ - the uncertainty associated with the measurement of diameter 2 = ± 0.1 mm - rectangular distribution

$\boldsymbol{u}_l$ - the uncertainty associated with the effect on load due to the load rate variation allowed in the test method = ± 0.025 MPa

Step 4: Calculating Standard Uncertainties

Standard uncertainties are calculated by dividing the uncertainties by the factor corresponding to the distribution used.

See the attached spreadsheet, column I shown in green.

Step 5: Weighting

The values of weighting for the load uncertainties were calculated using small increments in load, for the diameter small increments of diameter.

As the load rate directly affects the compressive strength, its weighting is 1.

See the attached worksheet, column J in turquoise.

Step 6: Calculations

For each uncertainty component calculate the final value ($\boldsymbol{u}_i\boldsymbol{c}_i$) by multiplying the standard uncertainty $\boldsymbol{u(x)}_i$ by the weighting factor $\boldsymbol{c}_i$. Shown in blue in Column K.

Square these values of ($\boldsymbol{u}_i\boldsymbol{c}_i$) to express them in terms of a variance $\boldsymbol{u}_i\boldsymbol{(y)}^2$. Shown in blue in Column L.

Add the individual values of the variance value for each component to give the sum of the variances $\boldsymbol{\Sigma u_i(y)^2}$. Shown in blue in Cell L25.

Calculate the square root of the sum of the variances of each component ***u(y)*** which is the combined uncertainty for the test result. Shown in blue in Cell L26.

Step 7: Expanded Uncertainty (Mu) and Report

A coverage factor of 2 is selected for the rigour required for this type of test. The combined uncertainty calculated in Step 6 is multiplied by the coverage factor to give the expanded uncertainty. Finally the test result and uncertainty value to be reported are shown on the spreadsheet.

Source of uncertainty	Distribution	Uncertainty	Factor	Standard uncertainty	Weighting	Calculations	Coverage factor	Expanded uncertainty	Report	Data Entry

Test Parameters	*Symbol*	*Test Data*	*Increment*	Weighting Calculations			
				P	d_1	d_2	π
Load Applied (kN)	***P***	193	1	194	193	193	193
Diameter of specimen (mm)	d_1	100.2	0.1	100.2	100.3	100.2	100.2
Diameter of specimen (mm)	d_2	100.2	0.1	100.2	100.2	100.3	100.2
PI	***Π***	3.142	0.001	3.142	3.142	3.142	3.143
Compressive Strength MPa	***UCS***	24.4723501		24.5991498	24.44794484	24.4479448	24.4645638
			Weighting	0.12679974	- 0.244052342	-0.2440523	-7.78630292

Measurement	*Source*	*Uncertainty*	*Distribution Type*		*U value*	*Dist Factor*	*Std Uncertainty* $u(x)_i$	*Weighting* c_1	u_ic_i	$(u_i(y))^2$
Applied Load (kN)	testing m/c cal	u_p	rectangular		1.93	1.73205	1.11428602	0.12679974	0.1412912	0.0199632
	Reading	u_m	rectangular		0.5	1.73205	0.28867513	0.12679974	0.0366039	0.0013398
	Eccentricity	u_e	rectangular		0.193	1.73205	0.1114286	0.12679974	0.0141291	0.0001996
	cap angle	u_{cap}	rectangular		0.06562	1.73205	0.03788572	0.12679974	0.0048039	0.0000231
	Slenderness	u_s	rectangular		0	1.73205	0	0.12679974	0.0000000	0.0000000
Diameter (mm)	Vernier	u_{d1}	rectangular		0.1	1.73205	0.05773503	-0.2440523	-0.0140904	0.0001985
Diameter (mm)	Vernier	u_{d2}	rectangular		0.1	1.73205	0.05773503	-0.2440523	-0.0140904	0.0001985
Constant	pi rounding	u_{pi}	rectangular		0.0004	1.73205	0.00023094	-7.7863029	-0.0017982	0.0000032
UCS (MPa)	Pacer	u_l	rectangular		0.025	1.73205	0.01443376	1	0.0144338	0.0002083
									SUM	0.0221344
									$u_c(y)$	0.1487763
									k	2
									U_{95}	0.30

REPORT:	**Compressive Strength**		**24.0**	MPa
	with an expanded measurement uncertainty of	**±**	**0.3**	MPa
	at a confidence level of		**95**	%
	with a nominal coverage factor of		**2**	

The uncertainty of measurement value shown does not include any estimate of the effects associated with sampling or field and lab curing. Test results should be assessed using precision in terms of repeatability and reproducibility, measurement uncertainty and effects of sampling and curing

Example 3 - Field Density of Soil - AS 1289.5.3.1

Step 1: What is being measured?

The nominated test procedure is AS 1289.5.3.1. Both wet and dry field density are determined. Moisture content of the soil (*w*) for this example is determined in accordance with AS 1289.2.1.1. The example also includes the performance of a surface correction at the site. An additional measurement uncertainty will need to taken into account if the surface correction measurement is not made.

Calibration of apparatus and density sand

The test method requires that calibration of the mass of sand in the cone and the pouring density of sand is performed. This is not done for each field test and the results of the calibrations and the measurement uncertainties should be maintained in a calibrations file. Details of the MU calculations are shown in Appendices A and B.

Wet and dry density of soil

Mass of sand to fill hole $\boldsymbol{m}_{12}$

$\boldsymbol{m}_{12} = (\boldsymbol{m}_9 - \boldsymbol{m}_{10}) - (\boldsymbol{m}_7 - \boldsymbol{m}_8)$ - Equation A

Where:

- $\boldsymbol{m}_9$ = mass of container filled with sand
- $\boldsymbol{m}_{10}$ = mass of remaining sand and container
- $\boldsymbol{m}_7$ = mass of container plus sand for correction
- $\boldsymbol{m}_8$ = mass of remaining correction sand and container

Density of wet soil $\boldsymbol{\rho}$

$\boldsymbol{\rho} = \boldsymbol{m}_{11} \times \boldsymbol{\rho}_{sand} / \boldsymbol{m}_{12}$ - Equation B

Where:

- $\boldsymbol{m}_{11}$ = mass of excavated soil
- $\boldsymbol{\rho}_{sand}$ density of sand
- $\boldsymbol{m}_{12}$ = mass of sand required to fill the hole $\boldsymbol{m}_{12}$

Dry density of soil $\boldsymbol{\rho}_d$

$\boldsymbol{\rho}_d = \boldsymbol{\rho} \times 100)/(100 + \boldsymbol{\omega})$ - Equation D

Where:

- $\boldsymbol{\omega}$ = moisture content of soil

Step 2: Sources of Uncertainty

Uncertainties due to determination of mass+A41

For each mass determination there will be an uncertainty $\mathbf{u}_{m(n)}$ associated with it due to uncertainties in weighing associated with the balance.

There is also an uncertainty associated with the determination of the moisture content of the soil $\boldsymbol{u}_{mc}$.

The method could permit some loss of moisture in the sample while excavating the hole and placing the moist soil in the container. These losses due to handling the sample have been designated $\boldsymbol{u}_{loss.}$

Step 3: Estimates of Uncertainty and Types of Distribution

For the purpose of this example, the uncertainty related to the measurement of mass is taken from the maximum permitted Limit of Performance of the balance shown in the test method, i.e. $\boldsymbol{u}_{m(n)} = \pm 5$ g where n is number of the mass determination being carried out. This value is from a rectangular distribution with a factor of square root 2.

Note: Alternatively a lower value of $u_{m(n)}$ can be derived from the limit of performance of the balance shown on the calibration report may be used.

The uncertainty for moisture content $\boldsymbol{u}_{(w)}$ (%) was obtained from the estimate of uncertainty ***u(y)*** shown in the moisture content estimate for AS 1289.2.1.1 (see example 1 in this series). This value can be considered to come from a normal distribution with a factor of square root 2 as it has been obtained from another uncertainty calculation.

The uncertainty related to the loss of moisture $\boldsymbol{u}_{loss}$ was estimated to be in the range ± 0.2% moisture. This could be anywhere within the range and thus a rectangular distribution with a factor of square root 3.

Incorporating these uncertainties into the equations:

Equation A for the mass of sand to fill the hole becomes:

$\boldsymbol{m}_{12} = (\boldsymbol{m}_9 \pm \boldsymbol{u}_{m(9)} - \boldsymbol{m}_{10} \pm \boldsymbol{u}_{m(10)}) - (\boldsymbol{m}_7 \pm \boldsymbol{u}_{m(7)} - \boldsymbol{m}_8 \pm \boldsymbol{u}_{m(8)})$ with an uncertainty of $\boldsymbol{u}_{m(12).}$

Equation B for wet density becomes:

$\boldsymbol{\rho} = (\boldsymbol{m}_{11} \pm \boldsymbol{u}_{m(11)}) \times (\boldsymbol{\rho}_{sand} \pm \boldsymbol{u}_{\rho sand}) / (\boldsymbol{m}_{12} \pm \boldsymbol{u}_{m(12)})$ with an uncertainty of measurement $\boldsymbol{u}_{\rho.}$

Combining equations A and B gives equation C

$\boldsymbol{\rho} = (\boldsymbol{m}_{11} \pm \boldsymbol{u}_{m(11)}) \times (\boldsymbol{\rho}_{sand} \pm \boldsymbol{u}_{\rho sand}) / ((\boldsymbol{m}_9 \pm \boldsymbol{u}_{m(9)} - \boldsymbol{m}_{10} \pm \boldsymbol{u}_{m(10)}) - (\boldsymbol{m}_7 \pm \boldsymbol{u}_{m(7)} - \boldsymbol{m}_8 \pm \boldsymbol{u}_{m(8)}) / (\boldsymbol{m}_{12} \pm \boldsymbol{u}_{m(12)})$ with an uncertainty of measurement $\boldsymbol{u}_{\rho.}$

Equation D for dry density becomes:

$\boldsymbol{\rho}_d = (\boldsymbol{\rho} \pm \boldsymbol{u}_\rho) \times 100)/(100 + (\boldsymbol{\omega} \pm \boldsymbol{u}_w \pm \boldsymbol{u}_{loss})$ with an uncertainty of measurement $\boldsymbol{u}_{\rho d}$.

Step 4: Calculating Standard Uncertainties

Standard uncertainties are calculated by dividing the uncertainties by the factor corresponding to the distribution used.

See the attached spreadsheet, column shown in green.

Step 5: Weighting

The values of weighting for the uncertainties were calculated using small increments in mass, pouring density and moisture content respectively. These are shown in columns F to K in turquoise and then used in the uncertainty calculation in Column I.

Step 6: Calculations

For each uncertainty component calculate the final input value ($\boldsymbol{u}_i\boldsymbol{c}_i$) by multiplying the standard uncertainty $\boldsymbol{u(x)}_i$ by the weighting factor $\boldsymbol{c}_i$. Shown in blue in Column J.

Square these values of ($\boldsymbol{u}_i\boldsymbol{c}_i$) to express them in terms of a variance $\boldsymbol{u}_i\boldsymbol{(y)}^2$. Shown in blue in Column K

Add the individual values of the variance value for each component to give the sum of the variances $\boldsymbol{\Sigma u_i(y)^2}$. Shown in blue in Cell K28 and K53.

Calculate the square root of the sum of the variances of each component $\boldsymbol{u(y)}$ which is the combined uncertainty for the test result. Shown in blue in Cell K29 and K54.

Step 7: Expanded Uncertainty (Mu) and Report

A standard coverage factor of 2 is selected for the rigour required for this type of test. The combined uncertainty calculated in Step 6 is multiplied by the coverage factor to give the expanded uncertainty. Finally, the test result and uncertainty value to be reported are shown on the spreadsheet.

Source of error	Distribution	Error	Factor	Standard uncertainty	Weighting	Calculations	Coverage factor	Expanded uncertainty	Report	Data Entry

FIELD WET DENSITY - Equation B

WEIGHTING CALCULATIONS

Test Parameters	*Symbol*	*Test Data*	*Increment*	*Weighting Calculations* m_7	m_8	m_9	m_{10}	m_{11}	ρ_{sand}
SC mass of sand in container	$m_{(7)}$	9000	1	9001	9000	9000	9000	9000	9000
SC mass of residual sand in container	$m_{(8)}$	7023	1	7023	7024	7023	7023	7023	7023
mass of sand in container	$m_{(9)}$	9000	1	9000	9000	9001	9000	9000	9000
mass of residual sand in container	$m_{(10)}$	1156	1	1156	1156	1156	1157	1156	1156
mass of excavated soil	$m_{(11)}$	8423	1	8423	8423	8423	8423	8424	8423
pouring density	ρ_{sand}	1.832	0.001	1.832	1.832	1.832	1.832	1.832	1.833
mass of sand in hole	m_{12}	5867		5866	5868	5868	5866	5867	5867
field wet density	ρ	2.6301237		2.63057211	2.629675528	2.629675528	2.63057211	2.630435998	2.6315594
			WD weighting	-0.000448367	-0.000448215	-0.000448215	0.000448367	3.70717E-08	1.4356571

UNCERTAINTY CALCULATIONS

Measurement	*Source*	*Uncertainty*	*Distribution Type*	*u Value*	*Divisor*	*Std Uncertainty* $u(x)_i$	*Weighting* c_i	$u_i c_i$	$(u_i(y))^2$
SC mass of sand in container	balance	$u_{m(7)}$	rectangular	5	1.732050808	2.886751346	-0.000448367	-0.001294325	1.675E-06
SC mass of residual sand in container	balance	$u_{m(8)}$	rectangular	5	1.732050808	2.886751346	-0.000448215	-0.001293884	1.674E-06
mass of sand in container	balance	$u_{m(9)}$	rectangular	5	1.732050808	2.886751346	-0.000448215	-0.001293884	1.674E-06
mass of residual sand in container	balance	$u_{m(10)}$	rectangular	5	1.732050808	2.886751346	0.000448367	0.001294325	1.675E-06
mass of excavated soil	balance	$u_{m(11)}$	rectangular	5	1.732050808	2.886751346	3.70717E-08	1.07017E-07	1.145E-14
pouring density **	calibration	u_{psand}	calculated			0.0080234	1.435657065	0.011518851	0.0001327
								SUM	0.000136
								u_ρ	0.0116633
									2
								U_{95} *WD*	0.0233267

FIELD DRY DENSITY -Equation D

WEIGHTING CALCULATIONS

Test Parameters	*Symbol*	*Test Data*	*Increment*	*Weighting Calculations*	
				ρ	ω
wet density	ρ	2.6301237	0.01	2.640123743	2.630123743
moisture content	ω	23	0.1	23	23.1
dry density	ρ_d	2.138312		2.146442067	2.136574933
			weighting	0.81300813	-0.017370528

FIELD DRY DENSITY UNCERTAINTY CALCULATIONS

Measurement	*Source*	*Uncertainty*	*Distribution Type*	*u Value*	*Divisor*	*Std Uncertainty*	*Weighting*		
						$u(x)_i$	c_i	$u_i c_i$	$(u_i(y))^2$
field wet density (t/m^3)	calculated	u_ρ				0.011663333	0.81300813	0.009482385	8.992E-05
moisture content (%)	calculated*	u_ω				0.4720432	-0.017370528	-0.00819964	6.723E-05
moisture losses (%)	handling	u_{loss}	rectangular	0.2	1.732050808	0.115470054	-0.017370528	-0.002005776	4.023E-06
								SUM	0.0001612
								$u_{\rho d}$	0.0126954
								k	2
								U_{95} *DD*	0.0253908

* taken from example of moisture content in this series of examples

REPORT:	**Field Wet Density**		**2.63**	t/m^3
	with an expanded measurement uncertainty of	**±**	**0.02**	t/m^3
	at a confidence level of		**95**	%
	with a coverage factor of		**2**	
	Field dry density		**2.14**	t/m^3
	with an expanded measurement uncertainty of	**±**	**0.03**	t/m^3
	at a confidence level of		**95**	%
	with a coverage factor of		**2**	

The uncertainty of measurement value shown does not include any estimate of the effects associated with sampling. Test results should be assessed using precision in terms of repeatability and reproducibility, measurement uncertainty and sampling effects

APPENDIX A
VOLUME OF CONTAINER

Test Parameters	*Symbol*	*Test Data*	*Increment*	*Weighting Calculations*		
				m_c	m_w	ρ_w
Mass of container (g)	m_c	1511	1	1512	1511	1511
Mass of container filled with water (g)	m_w	2689	1	2689	2690	2689
Density of Water (t/m^3)	ρ_w	0.99973	0.00001	0.99973	0.99973	0.99974
Volume of container (mL)	V	1177.6819		1176.68221	1178.68167	1177.69372
			Weighting	-0.99973	0.99973	1178

VOLUME OF CYLINDER UNCERTAINTY CALCULATIONS

Measurement	*Source*	*Uncertainty*	*Distribution Type*	*u Value*	*Divisor*	*Std Uncertainty* $u(x)_i$	*Weighting* c_i	$u_i c_i$	$(u_i(y))^2$
Mass of container (g)	balance	u_{mc}	rectangular	5	1.732050808	2.886751346	-0.99973	- 2.885971923	8.3288339
Mass of container filled with water (g)	balance	u_{mw}	rectangular	5	1.732050808	2.886751346	0.99973	2.885971923	8.3288339
Density of water (t/m^3)	thermometer	$u_{\rho w}$	rectangular	0.00027	1.732050808	0.000155885	1178	0.183632027	0.0337207
								SUM	16.691389
								u_V	4.0855096
								k	2
								U_{95}	8.1710192

REPORT:	**Volume of Container**	**1178**	mL
	with an expanded measurement uncertainty of	**± 8**	mL
	at a confidence level of	**95**	%
	with a coverage factor of	**2**	

APPENDIX B
POURING DENSITY OF SAND

Test Parameters	Symbol	Test Data	Increment	Weighting Calculations				
				m_1	m_2	m_4	m_5	V
Mass to fill cone (g)	m_1	9000	1	9001	9000	9000	9000	9000
Mass of remaining sand (g)	m_2	7232	1	7232	7233	7232	7232	7232
Mass of sand remaining in container (g)	m_4	4432	1	4432	4432	4433	4432	4432
Mass of sand + container (g)	m_5	8358	1	8358	8358	8358	8359	8358
Volume (mL)	V	1177.6819	1	1177.6819	1177.6819	1177.6819	1177.6819	1178.6819
Pouring density (t/m3)	ρ_{sand}	1.8324133		1.831564194	1.833262445	1.831564194	1.833262445	1.830858691
			Weighting	-0.000849126	0.000849126	-0.000849126	0.000849126	-0.001554629

POURING DENSITY UNCERTAINTY CALCULATIONS

Measurement	Source	Uncertainty	Distribution Type	u Value	Divisor	Std Uncertainty $u(x)_i$	Weighting c_i	u_ic_i	$(u_i(y))^2$
sand + container(g)	balance	u_{m4}	rectangular	5	1.732050808	2.886751346	-0.000849126	-0.002451215	6.008E-06
sand in container (g)	balance	u_{m5}	rectangular	5	1.732050808	2.886751346	0.000849126	0.002451215	6.008E-06
Mass to fill cone (g)	balance	u_{m1}	rectangular	5	1.732050808	2.886751346	-0.000849126	-0.002451215	6.008E-06
Mass of remaining sand (g)	balance	u_{m2}	rectangular	5	1.732050808	2.886751346	0.000849126	0.002451215	6.008E-06
Volume (mL)	calibration	u_V	normal			4.0855096	-0.001554629	-0.006351453	4.034E-05
								SUM	6.437E-05
								$u_{\rho sand}$	0.0080234
								k	2
								U_{95}	0.0160468

REPORT	**Sand Pouring Density**	**1.83**	t/m^3
	with an expanded measurement uncertainty of	**± 0.02**	t/m^3
	at a confidence level of	**95**	%
	with a coverage factor of	**2**	

MULTIPLE DETERMINATIONS FOR POURING DENSITY

Enter the values obtained from each determination

ESDM is the standard deviation divided the square root of the number of determinations

It may be used when the method prescribes repeated measurements

The value estimated above from considering all sources of uncertainty errors are shown next to the ESDM values for comparison.

Note: Differences due to balances are more precise than the LoP of 5g used in the MU estimate.

POURING DENSITY

1	1.825	t/m^3	
2	1.830	t/m^3	
3	1.837	t/m^3	
Mean	1.8306667	t/m^3	
Standard Deviation	0.0060277	t/m^3	
ESDM	0.0034801	t/m^3	0.01